# 복잡한 물체의 기술을 위한 원뿔 표면의 분류 및 임계치 선정

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#### S 약

본 논문에서는 컴퓨터 시각에서 가장 중요한 과제의 하나인 3차원 물체의 표현에 대해 원뿔형태의 기술과 표면 분류시 임계치를 자동으로 선정하는 방법에 대해 제안하고자 한다. 기존에 미분기하학에서 사용한 평균 곡률 (H)과 가우스 곡률(K)은 물체의 상당 부분을 차지하고 있는 원뿔표면에 대한 분류가 불가능하였다. 또한 평균 곡률과 가우스 곡률의 부호값에 따른 표면 분류가 실제 거리 영상에 적용시 올바로 분류가 안 되는 문제를 가지고 있었다. 이 논문에서는 기존의 이 같은 두 가지 문제를 해결하기 위해 리지와 밸리의 표면분류로부터 원뿔표면 형태 (cone ridge, cone valley)를 분류해 내었다. 즉, 원통표면 형태의 경우 H값이 일정하고, 원뿔표면 형태의 경우는 H의 값이 다름을 이용하여 원 뿔표면 형태를 분류하였다. 아울리 통계적인 관점에서 표면분류 임계치를 선정할 수 있는 방법을 제안하고 실험에 의해 제안한 방법의 유용성 을 입증하고자 한다.

## Cone Surface Classification and Threshold Value Selection for Description of Complex Objects

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#### **ABSTRACT**

In this paper, the 3-D shape description for the objects with the cone ridge and valley surfaces, and the corresponding threshold value selection for surface classification are considered. The existing method based on the mean and Gaussian curvatures(H and K) of differential geometries cannot properly describe cone primitives, which are some of the most common objects in the real world. Also the existing method for surface classification based on the sign values of H and K has problems in practical applications. For this, cone surface shapes are classified: cone ridges and cone valleys are derived from surfaces using the fact that H values are constant in case of cylinder surfaces and variable for cone surfaces, respectively. Also threshold value selection for surface classification from a statistical point of view is proposed. The effectiveness of the proposed methods are verified through experiments.

키워드: 미분기하(Differential Geometry), 곡물(Curvature), 임계치(Threshold)

#### 1. Introduction

Recent research on computer vision has been focused more on practical commercial applications than on the theoretical side of the problem[1-5], [12-14]. Even though 2-D image processing techniques have been widely used in real world applications, still more work needs to be done on the 3-D computer vision for better accuracy and precision needed in practical industrial applications. Currently the 3-D computer vision problem is approached by three ways, i.e., active ranging[6] to acquire range data, passive ranging[7, 8] also known as shape from X, and 3-D object description and recognition process from acquired range data. Now, range data are used in many industrial applications including vehicle collision avoidance systems and numerous reasonably priced commercial products. However, the 3-D objects description requires more work to be done. The most widely applied methods for this problem now are 4 and 8-primitive methods using H and K curvatures of differential geometry[9]-[11]. The 4-primitive method is based on the fact that about 85 percent of the ordinary objects belong to these primitives, but the method is not applicable for the rest of the objects. The other method using 8 primitives describes objects more accurately. However this method fails in describing the

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cone surface objects which are very prevalent in everyday life. In the 8-primitive surface classification, using the sign values of H and K curvatures is logical, but the problem is to find an appropriate method to determine the sign values of H and K surfaces for the practical range data applications. Here we propose a method of selecting appropriate threshold values in H and K curvatures of differential geometry, which lead to a new technique in effective description of cone surfaces. Finally, the experiments are performed to prove the effectiveness of the proposed methods.

#### Surface Classification Using H, K Sign Values of Differential Geometry

Three-dimensional surface classification using H, K sign values of differential geometry is as follows:

Usually a surface, S(x, y, z) in 3-dimensional coordinate can be represented using the defined functions d, e, f with parameters, u and v.

$$s = \{(x, y, z) : x = d(u, v), y = e(u, v), z = f(u, v)\}$$
 (1)

Also, a curved surface can be defined using the first basic function and the second one of surface as follows:

The first basic function is equation (2).

$$I(u, v, du, dv) = dX \cdot dX = \begin{bmatrix} du \ dv \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$= du^{T} [g] du$$
(2)

where the elements of the matrix [g] is:

$$g_{11} = X_u \cdot X_u, g_{22} = X_v \cdot X_v, g_{12} = X_u \cdot X_v, = g_{21}$$
 (3)

And the second basic function is equation (4).

$$II(u, v, du, dv) = d_X \cdot d_N = [du \ dv] \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= du^T [b] du$$
(4)

where the elements of the matrix [g] is:

$$b_{11} = X_{uu} \cdot N, \ b_{22} = X_{vv} \cdot N, \ b_{12} = X_{uv} \cdot N$$
 (5)

and

$$N = \frac{X_u \times X_v}{|X_u \times X_v|} \tag{6}$$

And a shape operator,  $\beta$ , can be defined from the matrices, [g] and [b], in the first and second basic functions, respectively, as follows:

$$[\beta] = [g^{-1}][b]$$
 (7)

Then the mean curvature (H) and Gauss curvature (K) are as equations (8) and (9).

$$K = det[\beta] \tag{8}$$

$$H = 1/2 tr[\beta] \tag{9}$$

The surface classification is, then, made using the Gauss curvature (K) and the mean curvature (H) as in  $\langle Table 1 \rangle$ .

⟨Table 1⟩ Surface classification by the K and H sign values

К	-	0	+
_	Peak	Ridge	Saddle Ridge
0	(none)	Flat	Minimal Surface
+	Pit	Valley	Saddle Valley

But this method cannot describe the cone surface as can be seen in <Table 1>. Also the range of the K and H sign values must be selected when applying practical actual range data.

#### 3. Representation of Cone Surface

Firstly, to classify the cone surface type, we will prove the cone surface to belong to the ridge and the valley surface: The relationship of the principal curvatures, H curvature and K curvature, is as follows.

$$H = (k_1 + k_2) / 2 \tag{10}$$

$$K = k_1 \cdot k_2 \tag{11}$$

The equation of the cone ridge is equation (13).

$$f(x, y) = Z = \sqrt{x^2 - y^2}, (x > y)$$
 (13)

and

$$f_x = \frac{x}{R} \tag{14}$$

where  $R = \sqrt{x^2 - y^2}$ Also

$$f_{xx} = \frac{-y^2}{S} \tag{15}$$

where  $S = (x^2 - y^2)^{3/2}$ .

And

$$f_{y} = \frac{-y}{R} \tag{16}$$

$$f_{yy} = \frac{-x^2}{S} \tag{17}$$

$$f_{xy} = \frac{xy}{S} \tag{18}$$

Also

$$f_{xx}f_{yy} - f_{xy}^{x} = (-y^{2}/S) * (-x^{2}/S) - (xy/S)^{2}$$
 (19)

$$= x^2 y^2 / S^2 - x^2 y^2 / S^2$$
 (20)

$$= 0 \tag{21}$$

And

$$f_{xx} + f_{yy} + f_{xx} f_y^2 + f_{yy} f_x^2 - 2f_x f_y f_{xy}$$
 (22)

$$= (-y^2/S) - (x^2/S) + (-y^2/S) * (y^2/R^2) + (-x^2/S) + (x^2/R^2) - 2(-xy/R^2) * (xy/S)$$

$$= -\{(x^2 + Y^2)/S + (x^2 - y^2)^2/(R^2S)\}, (x > y) < 0$$
(23)

Also the equation of cone valley,

$$f(x, y) = Z = -\sqrt{x^2 - y^2}, (x > y)$$
 (24)

can be applied using the same method, and can be proved to belong to valley as H > 0, K = 0.

Hence, the classification of 10 surface types is now possible unlike the existing classification of 8 surface types in the present differential geometry. In another words, the addition of classification of cone ridge and cone valley surface types solve the problem of cone surface classification in the existing differential geometry.

### Practical Selection of Threshold Values for H, K Sign Values

#### 4.1 Relation between H and K

To find the relationship between mean and Gaussian curvatures, the flat surface from the range image is combined with the Gaussian noise value. Since both curvatures are of parabola, the linear relationship is established as the first step.

Then they are represented in terms of minimum and maximum values ( $k_1$  and  $k_2$ ) as shown in equations (27) and (28).

$$K = k_1 \cdot k_2 \tag{25}$$

$$H = (k_1 + k_2)/2 (26)$$

$$k_1 = H + \sqrt{H_2 + K} \tag{27}$$

$$k_2 = H - \sqrt{H_2 + K} \tag{28}$$

For  $k_1$  and  $k_2$  be real, K' should be negative. Both ex-

pressions show that 0.00005 is subtracted from K, which is enough to make the result of the expressions real since the experimental image depends on the curvature values from the range image.

$$K' = K - 0.00005 \tag{29}$$

To make the left and right densities equal, axis revolution by expression (30) is made to  $k_1$  and  $k_2$ .

$$X_5 = (k_1 - k_2)/2 \tag{30}$$

$$X_6 = k_2 \tag{31}$$

 $X_5$  and  $X_6$  here are mutually independent as their relation is graphically shown in (Figure. 1).

The needed joint probability density function of  $X_5$  and  $X_6$  is as follows:

$$f(X_5, X_6) = f(X_5)f(X_6)$$

$$= aEXP[-\{a(X-b)(2\pi a^2)\}^{-\frac{1}{2}}[EXP\{-(X_6-\mu)/\sigma\}^2/2]$$

$$= a(2\pi\sigma^2)^{-\frac{1}{2}}EXP[-\{(X_6-\mu/\sigma)^2/2 - a(X_5-b)\}]$$
(32)

The range of parameter values confined by transformed  $X_5$  and  $X_6$  is as follows  $(a > 0, \sigma > 0, -\infty < b < \infty, 0 < \mu < \infty, b < X_5 < \infty, -\infty < X_6 < \infty).$ 

$$Y = a(X_5 - b) \tag{33}$$

$$Z = (X_6 - \mu) / \sigma \tag{34}$$

Now the joint probability density function of Y and Z are as follows  $(0 \le Y \le \infty, -\infty \le Z \le \infty)$ .

$$f(Y, Z) = (2\pi)^{-\frac{1}{2}} EXP\left\{-\frac{Z^2}{2} + Y\right\}$$
 (35)

#### 4.2 Extraction of Simultaneous Confidence Interval

The confidence interval for two curvatures is obtained by getting the interval that the joint probability densities are equal. When  $(\mathbb{Z}^2/2 + \mathbb{Y})$  becomes constant C, then  $f(\mathbb{Y}, \mathbb{Y})$ 

#### Z) is constant.

Accordingly the confidence interval becomes the closed interval between  $Y = -Z^2/2 + C$  and Y = 0 where the value of C is decided as in expression (35). Now the false alarm value is established to decide the constant C to form the size of the closed region.

$$P_r(H_0 \mid H_1, Y = 0 \text{ and } Z = 0)$$
 (36)

The above expression represents  $100(1 - \alpha)\%$  confidence interval.

To get the value of C is done by fixing the value of and double integrate f(Y,Z) on Y and Z, so that the value of the result is 1-  $\alpha$ .

$$P_{r}\{(Z^{2}/2 + Y) < C, Y > 0)\}$$

$$= \int_{0}^{\sqrt{2C}} \int_{0}^{C\frac{Z^{2}}{2}} f(Y, Z) dy dz$$

$$= \int_{0}^{\sqrt{2C}} \int_{0}^{C\frac{Z^{2}}{2}} (2\pi)^{-\frac{1}{2}} EXP\{(-Z^{2}/2 + Y)\} f(Y, Z) dy dz$$

$$= 2 \Psi\{(2C)^{-\frac{1}{2}} - 2(C/\pi)^{-\frac{1}{2}} EXP(-C) - 1\}$$

$$\psi(t) = 0.5 + 0.5\{1.0 - EXP(-2t^{2}/\pi)\}$$

$$\{1 + 2(\pi - 3)t^{4}/(3\pi^{2})\}^{\frac{1}{2}}$$
(38)

Here  $\Psi(\cdot)$  is the standard normal distribution function. The evaluation of the function  $\Psi(\cdot)$ , expression (37) is substituted with expression (38). Now, the following inequality is established, in which by considering the maximum value of C to 0, practical surface classification is possible. The method shown above, make the practical classification of the surfaces possible by automatically computing the C values satisfying the expression (39). Previously the classication is done by manually setting the threshold values in relation to physically verifying the images and the classification results.

$$\{1.0 - \text{EXP}(-4C/\pi)(1 + 8(\pi - 3)C^4/(3\pi_2))\}^{\frac{1}{2}}$$

$$-\left(\frac{C}{\pi}\right)^{\frac{1}{2}} \text{EXP}(-C)\langle (1 - \alpha) \rangle$$
(39)

#### 5. Experimentation and Consideration

The experiments in this paper are performed using C language on IBM-PC. Firstly, the experimental results are given to show the possibility of classification of cone surface type. (Figure 2) and (Figure 7) are the graphical processing results of range data for objects of various

surface types. Figs. (Figure 3) and (Figure 8) show the result of surface classification for Figs. (Figure 2) and (Figure 7). We can see the surface classification has been successfully performed on the cone surface. Also the experimental results of threshold value selection for H, K values are given in Figs. (Figure 4)  $\sim$  (Figure 6) and Figs. (Figure 9)  $\sim$  (Figure 11). Firstly, Figs. (Figure 4) and (Figure 9) are the input range data, and Figs. (Figure 5)  $\sim$  (Figure 6), (Figure 10)  $\sim$  (Figure 11) are the results of surface classification.

It can be seen that there is no difference between the results in Figs. (Figure 5) and (Figure 6). However, (Figure 5) is the best result manually performed by the experimenter while observing the intermediate results on the monitor. (Figure 6) is the automatically processed result using the proposed method in this paper. This means the proposed method has considerably better technical virtues in the context of implementations sake.

Also the surface classification result in (Figure 11) is considered to have a considerable improvement in the same context, although the results in both (Figure 10) and (Figure 11) do not show significantly visible differences.

Future work will seek to address the issues of threshold values selection of variousimage processing techniques as well as surface classification.

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#### 6. Conclusion

This paper opens an easy way in a method for the classifying the cone ridge surface and cone valley surface. Also a way to automatically classify the surfaces is presented.

Further research is to extend the practical threshold value selection method to the overall image processing and understanding.

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